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★**Introduction to real analysis.**

Graduate Texts in Mathematics, 280.
Springer, *Cham*, 2019. *xvii*+400 *pp. ISBN* 978-3-030-26901-2; 978-3-030-26903-6

This book gives an accessible introduction to real analysis that is suitable for first-year graduate students. It focuses on measure theory for Lebesgue measure in Euclidean spaces. Topics include Lebesgue measure on $\mathbb{R}^d$, the Lebesgue integral, differentiation and absolute continuity, L^p spaces, the Hilbert structure of L^2, and applications to studying convolutions and the Fourier transform.

This book is written in a clear style that is suitable for students reading on their own or as part of a guided class. Necessary discussion and definitions are clearly given. Results and their proofs are complete and correct. The author consistently motivates why the content is important and what the key ideas are. The priority is not to develop the material in the shortest amount of time, but rather to explain the material carefully in a way that students can follow. Diagrams are included where helpful, such as in illustrating the relationships between the different types of convergence of functions. The book has a great number of problems for students to work on, both throughout the text and at the end of each section. The author also makes available online various supplemental materials, including extra chapters and an instructor's guide.

Chapter 1 gives a review of important material on metric and normed spaces. Chapter 2 then introduces exterior Lebesgue measure, measurable sets, and Lebesgue measure in $\mathbb{R}^d$. Chapter 3 describes measurable functions whose values are either extended real numbers or complex numbers. This includes Egorov's and Lusin's theorems and a discussion of the relationship between different types of convergence. Chapter 4 develops the Lebesgue integral with respect to Lebesgue measure on $\mathbb{R}^d$, including the monotone and dominated convergence theorems and the relationship with the Riemann integral. Fubini's and Tonelli's theorems in this context are also discussed.

Chapter 5 begins the study of differentiation. It describes the Cantor function, followed by functions of bounded variation, differentiability of monotone functions, and the Lebesgue differentiation theorem. Chapter 6 concludes the study of differentiation by studying absolute continuity and the generalization of the fundamental theorem of calculus. This includes a discussion of growth lemmas, which give conditions where one can bound the measure of the image of a measurable set. Chapter 7 introduces L^p and l^p spaces, including the key inequalities and properties like density of continuous functions with compact support.

Finally Chapters 8 and 9 cover extra material not always included in a first course on real analysis. Chapter 8 gives an introduction to Hilbert spaces and the inner product structure of L^2; then Chapter 9 studies convolution and the Fourier transform.

In summary, this book gives an accessible introduction to real analysis with emphasis on Lebesgue measure and Lebesgue integration in Euclidean spaces. This book could be suitable as a primary text for a first course focused on measure theory in Euclidean spaces or, due to the numerous exercises throughout, as a supplemental text for instructors giving other introductory measure theory courses. *Gareth Speight*