Zbl
Heil, Christopher
A basis theory primer. Expanded ed.

Applied and Numerical Harmonic Analysis. Basel: Birkhäuser (ISBN 978-0-8176-4686-8/hbk; 978-0-8176-4687-5/ebook). xxv, 534 p. EUR 69.50 (2011).

This book is a very comprehensive work dedicated to introducing graduate students or researchers in pure and applied mathematics as well as engineering to the foundations of basis expansions and to essential techniques for applications. Since Fourier studied series expansions in trigonometric polynomials to solve the heat equation, bases, Banach spaces and approximation strategies are at the heart of many problems in pure and applied mathematics. Chris Heil's book explains many aspects of Fourier's legacy which go beyond harmonic analysis, including Banach space geometry, nonharmonic Fourier analysis, and the modern theory of frames. The exercises contained in the book make it a good fit for graduate courses on selected topics in functional analysis and applications.

The first half of the book, Chapters 1-7, develops bases and frames from an abstract viewpoint. Standard results in Banach space theory and in Hilbert spaces are reviewed. These results are then incorporated in a study of convergence results of infinite series in Banach spaces with respect to different topologies. Unconditional and the generally stronger notion of absolute convergence are compared. This discussion leads to basis expansions, with concrete examples for bases such as the Schauder, Haar or the trigonometric systems. The distinction between bases and exact systems is explained, the former associated with unique expansions, whereas the latter are merely minimal systems with a dense linear span. The insights offered here are further developed into a discussion of stability for basis expansions. This topic is crucial in many applications.

The second half of the book, Chapters 8-14, emphasizes the role of specific types of bases and frames in pure and applied harmonic analysis. The introduction of frames drops the assumption of minimality for expansions, while retaining many properties such as stability and unconditional convergence. Gabor systems and wavelets illustrate these concepts, with relevance for applications in signal processing. Recent results in the context of Gabor frames are also explored, from Wiener amalgam spaces to the Zak transform and the Balian-Low theorem. Even an unresolved problem about the linear independence of finite time-frequency shifts of a non-zero vector in $L^2(R)$ is presented, the so-called HRT conjecture of *C. Heil, J. Ramanathan* and *P. Topiwala* ["Linear independence of time-frequency translates," Proc. Am. Math. Soc. 124, No.9, 2787–2795 (1996; Zbl 0859.42023)]. Thus, the book completes a tour from the fundamentals of functional analysis to currently active research related to basis expansions. Finally, Chapters 13 and 14 return to classical results on convergence of Fourier series in L^p-spaces, for which the material on unconditionally convergent expansions is complemented by traditional tools in harmonic analysis.

Many topics in Chris Heil's book are also covered by [*O. Christensen*, An introduction to Frames and Riesz Bases. Applied and Numerical Harmonic Analysis. Boston, MA: Birkhäuser. (2003; Zbl 1017.42022)] but the level of detail found here is greater. For additional material on time-frequency analysis, see the text by [*K. Gröchenig*, Foundations of Time-Frequency Analysis. Applied and Numerical Harmonic Analysis. Boston, MA: Birkhäuser. (2001; Zbl 0966.42020)]. *Bernhard Bodmann (Houston)*

Classification: 42-02 46-01 43-02 42C15 42C20 42C30
Keywords: Bases; Banach spaces; Hilbert spaces; Schauder bases; Riesz bases; unconditional bases; frames; Fourier series; wavelets; Gabor systems; Wiener amalgam spaces; Zak transform; Balian-Low theorem
doi:10.1007/978-0-8176-4687-5