BOOK REVIEW

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A First Course in Fourier Analysis. *By David W. Kammler*. Prentice Hall, Upper Saddle River, NJ, 2000. $95.50. xv+790 pp., hardcover. ISBN 0-13-578782-3.

Fourier analysis is a beautiful, deep, and wide-ranging subject, with applications and connections throughout mathematics and the sciences. Before seeing this text, I gathered that Kammler's book intended to present both the mathematics of Fourier analysis and a number of its applications in a form accessible to undergraduates. I approached this claim with considerable skepticism, expecting that those "undergraduates" would turn out to be required to have backgrounds in real analysis, measure theory, complex analysis, functional analysis, differential equations, and maybe algebra and topology as well. To my surprise, Kammler has indeed written a text that is accessible to somewhat more typical mathematics, science, and engineering undergraduates. The majority of the text should be accessible (not necessarily *easily*) to students who have fluency with first-year calculus and undergraduate linear algebra and have a working knowledge of complex-number arithmetic. While some content choices obviously had to be made, the text covers a good deal of both theory and applications. To give my summary judgment now, on the whole I was pleased and impressed with the text, and I encourage potential instructors to consider it.

The text is divided into two parts. The first half is "Mathematics." Five chapters present the basics of Fourier transforms, convolutions, calculations of Fourier transforms, and related material. Chapter 6 presents the fast Fourier transform (FFT), one of the most important algorithms of modern times, and Chapter 7 is devoted to generalized functions (distributions). The second half contains "Selected Applications," with one chapter each on sampling theory, partial differential equations, wavelets, music, and probability. This is a lot of material to cover, and indeed the text runs a hefty 790 pages plus appendices and other material. Not all of this can be presented in a semester course, so the instructors will need to decide which topics and applications they want to cover.

One way that the text is made accessible to undergraduates is by restricting the types of functions that are considered. For most of the book, only piecewise continuous functions appear. This avoids the need for extensive use of measure theory, but some of the beauty of the subject, and many potential topics, is lost in the process. Further, results are not given the "most rigorous" presentation, but I found the level of detail adequate and in line with the goals presented in the preface. As the author points out there, "For every hour that you spend presenting 19th-century advanced calculus arguments, however, you will have one less hour for explaining the 20th-century mathematics of generalized functions, sampling theory, wavelets, ..."

The first five chapters, presenting the basic mathematics, are written in considerable detail, with many calculations of actual Fourier transforms and Fourier series. The author's presentation is very complete in the sense that he develops simultaneously the theory of the Fourier transforms of functions on the real line $\mathbf{R}$, the torus $\mathbf{T}_p$ (congruent to $[0, p)$ under addition mod p), the integers $\mathbf{Z}$, and the group $\mathbf{P}_N$ of integers mod N ($\mathbf{P}$ for "polygon"). Each of these different transforms has its appropriate use, and it is difficult to find a text that considers all of them. That can certainly be viewed as an advantage, but it does mean that most formulas and many calculations are repeated four times. There is much concreteness here, which is good for undergraduates—we really do see how to compute Fourier transforms, convolutions, etc., and in great detail. Moreover, the clarity of exposition is quite good, but the multiplication of notation and terminology resulting from considering all four transforms together makes these chapters cluttered and possibly a little difficult for inexperienced readers to follow. For me, it would be too tedious to present five full chapters worth of calculations in class, but this emphasizes again that the instructor will need to make some choices. There is certainly plenty to choose from.

After these initial chapters, I found the chapter on generalized functions quite nice, more elegant mathematically, perhaps, than the previous sections. Normally, tempered distributions are presented as continuous linear functionals on the Schwartz space. Since the Schwartz space is only a Frechet space and not a Banach space, a rigorous development via this approach requires rather intricate functional analysis. Instead, the author first presents the Schwartz class, then motivates in a reasonable way the idea of a generalized derivative, and presents tempered distributions as generalized derivatives of continuous, slowly growing functions. This is an unusual subject for an undergraduate text, with an unusual approach necessitated by the background limitations, but I believe I would enjoy presenting this chapter a great deal.

The remainder of the book contains five chapters on "Selected Applications." Of course, entire books exist on each of these chapters—dozens of books on wavelets alone have appeared in recent years. The treatment is not exhaustive. For example, the text portion of the chapter on sampling is 25 pages, and while quite a few nice results on reconstructing bandlimited functions from sample values are presented, there is no attempt to show how sampling theory arises in such real-life applications as computer-aided tomography. The chapter on music was very enjoyable. This is an unusual topic to find in a mathematics text and may be one that stimulates the interest of the students.

The wavelet chapter, at 100 pages, is the longest application chapter. The Haar wavelet system is presented first, and then the author does not hesitate to make extensive use of the Fourier analysis he has developed earlier to construct other wavelets, including the Daubechies wavelets. Surprisingly, however, this is not done by developing the theory of multiresolution analysis. Implementation via filter banks is developed, with a nice illustration of compressing a signature. As mentioned, this is not a complete survey of wavelet theory, but this chapter could be an excellent lead-in to a second semester devoted to that topic (with a similar remark applying to the other application chapters).

Each chapter is supplied with extensive exercise lists. Indeed, 28 of the 88 pages of Chapter 1 are devoted to exercises! The problems range widely in topic and depth and provide the instructor with a great deal of supporting material. The problems strike me as an extremely useful and valuable feature of this text.

In summary, this text contains a great deal of material, and the applications chapters in particular contain material that is difficult to find in one place in a form accessible to undergraduates (at least, those who have learned from the earlier chapters). The wealth of exercises may prove to be the best feature of all. The instructor will have to exercise care and think ahead about what material to present, but there is enough to choose from to satisfy the different inclinations of a wide variety of instructors and students.

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