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Metrics, Norms, Inner Products and Operator Theory: ERRATA

Updated July 5, 2021

Errata

1. p. 20, Item (4) of Definition 1.10.1: Replace "for all $x, y \in V$" with "for all $x, y, z \in V$".

2. p. 60, line 2. Replace "This implies that $\mathrm{d}(x, x_n) > r$ for every n," with "This implies that $\mathrm{d}(x, x_n) \ge r$ for every n,".

3. p. 70, 6 lines from bottom (last displayed equation of the proof of Theorem 2.6.8). Replace

$$\frac{1}{2} + \frac{1}{2} \;<\; 1 \qquad \text{with} \qquad \frac{1}{2} + \frac{1}{2} \;=\; 1.$$

4. p. 71, part (b) of Problem 2.6.14. Not exactly a typo—the problem statement is correct, but it is also possible for equality to fail if the index set I is finite.

5. p. 73, Problem 2.6.23. Change the definition of the set G_r from "$G_r(E) = \{x \in E : \mathrm{dist}(x, E) < r\}$" to "$G_r(E) = \{x \in X : \mathrm{dist}(x, E) < r\}$."

6. p. 81. Last two lines of the proof of the implication (c) $\Rightarrow$ (b). Replace "E" with "K".

7. p. 85. The proof of the implication "(c) $\Rightarrow$ (a)" of Lemma 2.9.4 contains an error. It is not sufficient to consider any point $x \in f^{-1}(V)$. Instead, we must use the assumption that $f^{-1}(V)$ is not open to select the appropriate point x to consider. Here is a corrected version of the proof of this implication.

(c) $\Rightarrow$ (a). Suppose that statement (c) holds, and let V be any open subset of Y. Suppose that $f^{-1}(V)$ were not open in X. Then there is some point $x \in f^{-1}(V)$ such that there is no radius $r > 0$ for which the open ball $B_r(x)$ is a subset of $f^{-1}(V)$. In particular, for each $n \in \mathbb{N}$ the ball $B_{1/n}(x)$ is not contained in $f^{-1}(V)$, and therefore there is some point $x_n \in B_{1/n}(x)$ such that $x_n \notin f^{-1}(V)$. As a consequence, $\mathrm{d}(x, x_n) < 1/n$ for every n, but $f(x_n) \notin V$ for any n.

Now, $x \in f^{-1}(V)$, so $f(x)$ does belong to V. Since V is open, there is some open ball centered at $f(x)$ that is entirely contained in V. That is, there is some radius $\varepsilon > 0$ such that $B_\varepsilon(f(x)) \subseteq V$.

On the other hand, we have $x_n \to x$, so by applying statement (c) we must have $f(x_n) \to f(x)$. Consequently, there is some $N > 0$ such that $\mathrm{d}\big(f(x), f(x_n)\big) < \varepsilon$ for all $n \geq N$. But then

$$f(x_N) \in B_\varepsilon(f(x)) \subseteq V,$$

which contradicts the fact that $f(x_N) \notin V$. Therefore $f^{-1}(V)$ must be open, and hence f is continuous.

8. p. 90, statement (a) of Problem 2.9.22. Change "for all $x, y \in X$" to "for all $x \neq y$ in X".

9. p. 118, 4 lines after Figure 3.3. Replace "also have $f_m(t) = f_n(t) = 0$ for" with "also have $f_m(t) = f_n(t)$ for".

10. p. 125, line 8 of the proof of Theorem 3.5.9. Change "satisfies $|x - y| < \delta$, then" to "satisfies $\mathrm{d}(x, y) < \delta$, then".

11. p. 194, Theorem 5.2.3. Remove extra comma in: "for all $x, y, \in H$."

12. p. 231, Figure 5.4. Change "$\cos(3t)$" to "$\cos(5t)$".

13. p. 235, Part (c) of Problem 5.10.12. Replace

$$\sum_{n=1}^{\infty} \frac{1}{(2n+1)^4} = \frac{\pi^4}{96} \qquad \text{with} \qquad \sum_{n=1}^{\infty} \frac{1}{(2n-1)^4} = \frac{\pi^4}{96}.$$

14. p. 236, Problem 5.10.14. Replace

$$\sum_{n=1}^{\infty} \frac{1}{(2n+1)^6} = \frac{\pi^6}{960} \qquad \text{with} \qquad \sum_{n=1}^{\infty} \frac{1}{(2n-1)^6} = \frac{\pi^6}{960}.$$

15. p. 250, Problem 6.2.5. Change "parts (b)–(c)" to "parts (b)–(d)".

16. p. 261, 12 lines from bottom. Replace "$Ax_n \to y$" with "$A_n x \to y$".

17. p. 266, line 7 of Example 6.6.6. In the definition of Tx, change "$x \in \ell^2$" to "$x \in H$".

18. p. 281, last line on the page. Replace "$\overline{c}\,T_y$" with "$\overline{c}\,T(y)$".

19. p. 282, Problem 6.8.5, part (a). Change "from A to ℓ^2" to "from H to ℓ^2".

20. p. 288, line 11. Change "$\diamondsuit$" symbol to the end-of-proof symbol "$\square$".

21. p. 289, 2 lines from bottom of page. Replace "$y \in \ell^p$" with "$y \in \ell^{p'}$".

22. p. 295. In the statement of Problem 7.1.5, change "for all $x, y \in H$" to "for all $x \in H$ and $y \in K$".

23. p. 302, line 5 of the proof of Theorem 7.2.7. Change "for every $z \in H$" to "for every $z \in K$".