

MATH 2171 Differential Equations

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The Ghostly Buick of Highway 21

It was a dark and stormy night. Officer James Ramirez sat in his patrol car listening to the sound of the rain hammering down onto the roof, the faint whisper of the police scanner the only interruption to the monotonous downpour. He flicked the ashes from his cigarette into a tray as he watched a bolt of lightning splinter the darkness. He knew it would be another lonely night camped out on route 21 looking for drunks and speed demons.

“Why do I always have to watch highway 21,” Ramirez thought to himself. It wasn’t that he disliked the night shift, or the long quiet hours, but this particular stretch of road had always carried an uneasiness for him. His thoughts drifted back to a happier time, a time when he still had his beloved wife. Anne was as intelligent as she was beautiful but as fate would have it she was taken from him on a night very much like this one, on a road very much like this one. The paramedics had said that her death would have been instant with no pain at all. If only he had been so lucky.

He reached down and took a sip of his coffee. He took it black, as black as the looming darkness just outside the glass of the car. Suddenly he realized the scanner had gone silent without his even noticing. When he looked up his eyes were met with two brilliant beams that he swore couldn’t have been there a moment before. The posted speed limit here was 35 mph (15 m/s) and these piercing lights were moving at least 50 mph (22 m/s) and still accelerating steadily. Having had the coffee in his hand and consuming thoughts in his head he was not able to get control of himself or the car until the speeding vehicle was about 80 meters away.

He pressed hard on the accelerator as he dropped the cup and all of its contents onto the passenger’s seat. Due to the drenched road though he was not able to keep the rear tires planted firmly on the road and the rear of the car swerved back and forth causing his velocity to take a form $v(t) =$

$$8\sqrt{t} + c \text{ (where } t \text{ is time in seconds).}$$

The fleeing automobile ahead of him appeared to be having just as much difficulty gaining traction and Ramirez figured the driver was drunk. Thinking back to his differential equations class he had so many years ago he guessed that the criminal's car had a velocity function of somewhere near

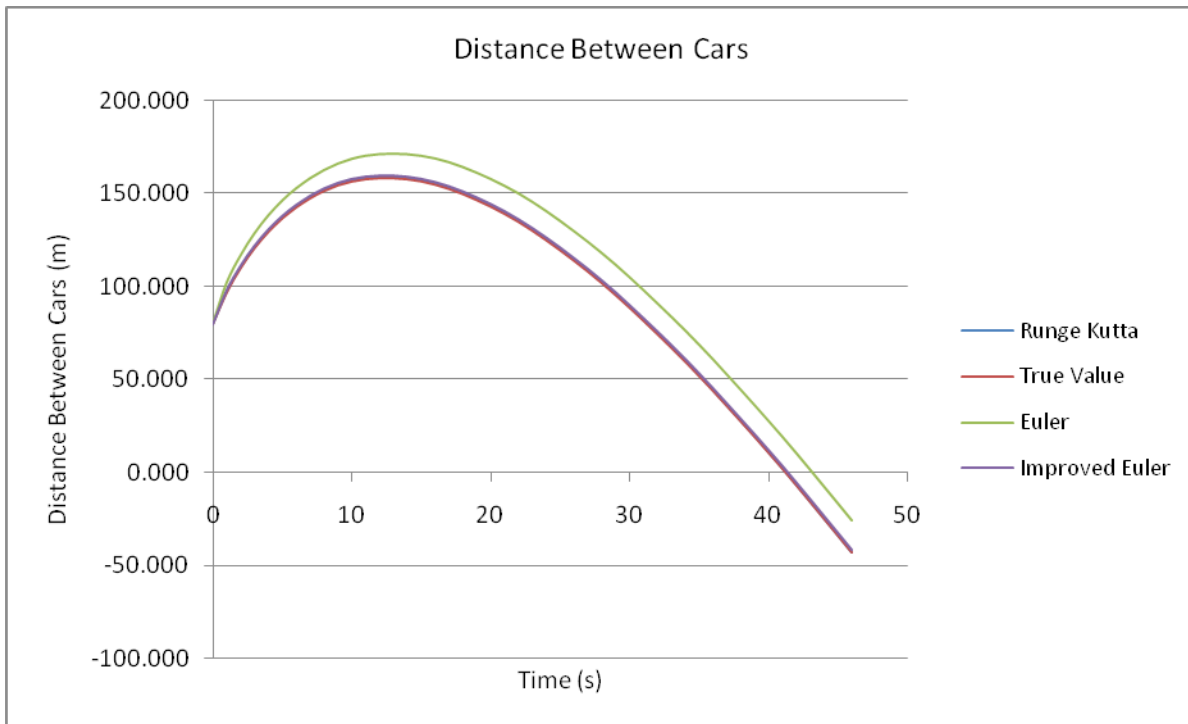
$$v(t) = \frac{t}{2} + \sin t.$$

As he pursued the car ahead he turned on the blue lights and blaring siren and called in the chase...no reply from the dispatch. He began to overtake the car and knew that it would only be a few more seconds until he reached his target.

He drew nearer still and wondered how many seconds it would take him from first setting off until this chase was ended and the violator caught. Having such complex equations governing the two bodies' velocities, a numerical approximation would probably be necessary. Ramirez's thoughts such as this were often quite eccentric in extreme situations.

"To numerically approximate the distance between our two cars the velocities would need to be subtracted to create a differential equation. The integral of this equation would yield a graph that would show the distance between our two vehicles versus time. The time until I reach the speeder would be when this distance becomes zero." Being very good at numerical approximation and having an artistic flare Ramirez was able to quickly jot down the values using the Euler method and graph it on a napkin. This gave him a value of somewhere between 43 and 44 seconds.

"The Euler method is very inaccurate though and I should probably try the Improved Euler and Runge-de-Kutta methods just to be sure," thought Ramirez. As his calculations and graphs showed the time was actually between 41 and 42 seconds.



Ramirez's napkin drawing.

"Damn," he thought, "I should have used smaller step sizes to get a better approximation."

He closed the final few meters between the cars and saw that it was a deteriorating Buick he was chasing. It had actually taken him about 41.262 seconds to get there but this was a value he would never come to know, for as he pulled alongside and glanced through his window at the occupant all his thoughts of differential equations disappeared.

How could it be? She had been dead for years yet he could not deny his own sight. She was there gazing back at him but it was as though she was an ocean away. He wanted to reach out and touch her, see if it was really her or just a figment of his unusual imagination but there was no time. The road curved off to one side and the final thoughts through Ramirez's mind as the patrol car tumbled down the ravine were of how the previous equations governing their velocity no longer applied...

