Christopher Heil

A Basis Theory Primer: Errata

Updated September 10, 2018

Errata

1. Page 10, in the line following Notation 1.11, replace "$3' = 4/3$" by "$3' = 3/2$".

2. Page 13, Exercise 1.11. This problem is only valid for indices p in the range $1 < p < \infty$.

3. Page 21, item (d) of Definition 1.25. Replace "$\{x_n\}$ is *complete*" with "S is *complete*".

4. Page 31, Exercise 1.30. Replace "$Ax \cdot x > 0$ for all $x \in \mathbf{F}^n$" with "$Ax \cdot x > 0$ for all nonzero $x \in \mathbf{F}^n$".

5. Page 54, final displayed equation (2 lines from bottom). Replace "$T(ax + by)$" with "$T(ay + bz)$".

6. Page 74, first line after equation (2.5). Replace "$\|x\|_X < r = s/2$" with "$\|x\|_Y < r = s/2$".

7. Page 74, last line of the proof of Lemma 2.26. Replace "$\|y\| < 1$" with "$\|z\| < 1$".

8. Page 99, line preceding equation (3.3). Replace "$F_0 \subseteq N$" with "$F_0 \subseteq \mathbf{N}$".

9. Page 100, Notation 3.12. In the definitions of $\mathcal{R}$, $\mathcal{R}_\mathcal{E}$, and $\mathcal{R}_\Lambda$, replace "$F \subseteq N$" with "$F \subseteq \mathbf{N}$".

10. Page 155, Theorem 5.6. Add hypothesis so that the integers n_k tend to infinity. Specifically, replace the first sentence of the theorem by "Let $0 \le n_1 \le n_2 \le \cdots$ be an increasing sequence of nonnegative integers *such that* $n_k \to \infty$."

11. Page 208, Example 8.6(e). The frame bounds of $\{2e_1, e_2, e_3, \dots\}$ are $A = 1$, $B = 4$ (not $A = 1$, $B = 2$). A different frame that has frame bounds $A = 1$, $B = 2$ is $\{e_1, e_1, e_2, e_3, \dots\}$.

12. Page 211, Conjecture 8.9. Not errata, but an update: The Kadison–Singer Conjecture has been proved! The proof is by A. Marcus, D. Spielman, and N. Srivastava in the paper "Interlacing Families II: Mixed Characteristic Polynomials and the Kadison-Singer Problem," 2013 (preprint available on the math arXiv). Casazza and Tremain had earlier proved that the Feichtinger Conjecture is equivalent to Kadison–Singer, therefore the Feichtinger Conjecture is now known to be true.

13. Page 220, Exercise 8.15. (a) We are implicitly assuming that U and V are nonnegative, and therefore $0 \le U \le V$.

    (b) Replace
    $$0 \;\le\; I - \frac{2}{A+B}\,S \;\le\; \frac{B-A}{B+A}\,I,$$
    by
    $$\frac{A-B}{B+A}\,I \;\le\; I - \frac{2}{A+B}\,S \;\le\; \frac{B-A}{B+A}\,I.$$
    The conclusion of the problem remains unchanged.

14. Page 229, last line of the proof of the "Rightarrow" direction of Corollary 8.30. Replace "UR" with "RU" twice, so that the line reads "Therefore $RU\colon H \to H$ is surjective and satisfies $RUe_n = x_n$."

15. Page 251, equation (9.4). Replace "$\frac{1}{r}\widehat{f}(\xi/r)$" with "$\frac{1}{r^{1/2}}\widehat{f}(\xi/r)$".

16. Page 257, 9 lines from bottom. Replace "$e_\xi(t)$" with $e_\xi(x)$".

17. Page 267, last line of the page. The independent variable on the left-hand side of the equation should be ξ instead of x. That is, replace "$d_\pi(x)$" with "$d_\pi(\xi)$".

18. Page 267, Example 10.3. In both the first and seventh line of the example, replace "$d_\pi(x)$" with "$d_\pi(\xi)$".

19. Page 267, Example 10.3. In the next to last line of the example, replace "Theorem 9.3" with "Theorem 9.5(a)".

20. Page 267, Example 10.3. In the last line of the example, the independent variable in the final term should be ξ, and a minus sign is missing in the exponent. That is, replace

$$e^{2\pi i a x}\chi_{[-\frac{1}{2},\frac{1}{2}]}(x)$$

with

$$e^{-2\pi i a \xi}\chi_{[-\frac{1}{2},\frac{1}{2}]}(\xi).$$

21. Page 275, equation (10.3). The independent variable on the right-hand side of this equation should be x instead of ξ. That is, replace

$$f(x) \;=\; b\sum_{n\in\mathbf{Z}} f(bn)\,\frac{\sin\pi(\xi-bn)}{\pi(\xi-bn)}$$

with

$$f(x) \;=\; b\sum_{n\in\mathbf{Z}} f(bn)\,\frac{\sin\pi(x-bn)}{\pi(x-bn)}.$$

22. Page 285, Definition 10.17. In part (b), a subspace is *translation-invariant* if it is invariant under *every real translate*. Therefore, in part (b) replace "for every $a \in \mathbf{Z}$" with "for every $a \in \mathbf{R}$".

23. Page 287, 4 lines from the bottom. The index for the series on this line is k and not n. That is, "$\sum_{n\in\mathbf{Z}} c_k\,T_k g$" should be replaced with "$\sum_{k\in\mathbf{Z}} c_k\,T_k g$".

24. Page 307, 6 lines from the bottom. A square is missing on the norm: replace $\|f\cdot T_{ak}\bar{g}\|_{L^2(I_k)}$ with $\|f\cdot T_{ak}\bar{g}\|^2_{L^2(I_k)}$.

25. Page 310, the Remark to Exercise 11.5. Not errata, but an update: X.-R. Dai and Q. Sun have solved this! They completely characterize the set of points (a,b) such that $\mathcal{G}(\chi_{[0,1]},a,b)$ is a frame in the paper "The *abc*-problem for Gabor systems," accepted for publication in the *Memoirs of the American Mathematical Society*. Also see their paper "The *abc*-problem for Gabor systems and uniform sampling in shift-invariant spaces," in: Excursions in Harmonic Analysis, Vol. 3, Birkhäuser/Springer, New York, 2015, pp. 177–194.

26. Page 312, line 2 of the proof of Corollary 11.7. The displayed equation is correct, but the second inequality can be replaced by an equality, so that it reads

$$Aab \;=\; \int_0^a Ab\,dx \;\le\; \int_0^a \sum_{k\in\mathbf{Z}} |g(x-ak)|^2\,dx \;=\; \int_{-\infty}^{\infty} |g(x)|^2\,dx \;=\; \|g\|^2_{L^2}.$$

27. Page 316, line 4. Replace $\mathcal{G}(g, 1, 1)$ with $L^2(\mathbf{R})$, so that it reads "it follows that $\mathcal{G}(g, 1, 1)$ is also an orthonormal basis for $L^2(\mathbf{R})$".

28. Page 316, Definition 11.12. Replace "consists of those functions $f \in L^p(\mathbf{R})$" with "consists of those functions $f\colon \mathbf{R} \to \mathbf{C}$".

29. Page 321, line 2 of the proof of Theorem 11.8. An "f" is missing in the series on the right-hand side. That is, the displayed equation should read

$$Lf \;=\; b^{-1} \sum_{n \in \mathbf{Z}} T_{\frac{n}{b}} f \cdot G_n$$

30. Page 321, line 4 of the proof of Theorem 11.8. A subscript L^2 is missing from the norm of Lf. That is, the displayed equation should read

$$\|Lf\|_{L^2} \;\le\; b^{-1} \sum_{n \in \mathbf{Z}} \|T_{\frac{n}{b}} f\|_{L^2} \, \|G_n\|_{L^\infty} \;\le\; B \, \|f\|_{L^2}$$

31. Page 323, line 8. Replace "Since T is bounded" with "Since L is bounded".

32. Page 331, Exercise 11.22. The first sentence of the exercise should read "If $f \in L^1(\mathbf{R}) = W(L^1, \ell^1)$ then $Zf \in L^1(Q)$ by Theorem 11.22."

33. Page 343, Conjecture 11.39. Conjecture should end with a "$\diamondsuit$" symbol.

34. Page 354, last line of Exercise 12.2. Replace "Heisenberg affine group" with "Heisenberg group".

35. Page 413, line 11. Replace "$[\widehat{f}, \widehat{g}\,]$" in the left-hand side of the displayed equation with "$[\widehat{f}, \widehat{\varphi}]$".

36. Page 437, Theorem 13.10. The statement of part (b) of this theorem should read as follows:

    (b) If c, $d \in \ell^1(\mathbf{Z})$ then $(c * d)^\wedge$ is the function

$$(c * d)^\wedge(x) \;=\; \widehat{c}(x)\, \widehat{d}(x).$$

37. Page 440, Definition 13.12. Replace "dt" in part (a) with "dx", so that the equation reads "$\int_0^1 k_N(x)\, dx = 1$ for every N".

Acknowledgment

We thank everyone who submitted typos! Many thanks to: Victor Bailey, Carlos Cabrelli, Yam-Sung Cheng, Barry Fell, Sung Ha Kang, Weilin Li, Anneliese Spaeth, and Nenad Teofanov.