

19. Some Extensions

Definition

Let (x_n) & (y_n) be sequences of real nos, $y_n \neq 0 \forall n$ large enough

a. (x_n) is equivalent to (y_n) , written $(x_n) \sim (y_n)$, if

$$\lim_{n \rightarrow \infty} \frac{x_n}{y_n} = 1.$$

b. (x_n) is a lower order of magnitude than (y_n) if

$$\lim_{n \rightarrow \infty} \frac{x_n}{y_n} = 0$$

Write $x_n = o(y_n)$

c. (x_n) is dominated by (y_n) if $(\frac{x_n}{y_n})$ is bounded:

$$\exists A, B \text{ st. } A \leq \frac{x_n}{y_n} \leq B \forall n. \quad \text{Write } x_n = O(y_n).$$

Switching Order

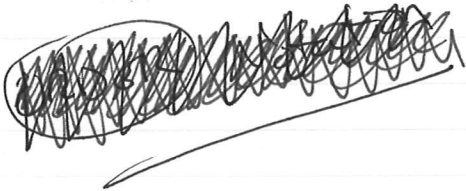
Can't switch limits, or sums & limits, or integrals & limits, in general.

Need not be true that

$$\lim_{n \rightarrow \infty} \lim_{m \rightarrow \infty} a_{m,n} = \lim_{n \rightarrow \infty} \lim_{m \rightarrow \infty} a_{m,n}$$

$$\sum_{n=1}^{\infty} \left(\lim_{n \rightarrow \infty} a_{m,n} \right) = \lim_{n \rightarrow \infty} \left(\sum_{m=1}^{\infty} a_{m,n} \right)$$

etc.



Fatou for series I-f $a_{m,n} \geq 0 \forall m,n$ then

$$\sum_m \liminf_{n \rightarrow \infty} (a_{m,n}) \leq \liminf_{n \rightarrow \infty} \left(\sum_m a_{m,n} \right)$$

Ex:

	1	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$...
	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{16}$...
Row m	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{16}$	$\frac{1}{32}$...
	\vdots	\vdots	\vdots	\vdots	

$$\rightarrow \liminf_{n \rightarrow \infty} a_{m,n} = 0 \rightarrow \sum_m \liminf_{n \rightarrow \infty} a_{m,n} = 0$$

↓
column n

$$\sum_m a_{m,n} = 2^{-n} \quad \liminf_{n \rightarrow \infty} \left(\sum_m a_{m,n} \right) = 0$$

But

	1	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$...
0	1	$\frac{1}{2}$	$\frac{1}{4}$		
0	0	1	$\frac{1}{2}$		
			1		

$$\rightarrow \liminf_{n \rightarrow \infty} a_{m,n} = 0 \quad \sum_m \liminf_{n \rightarrow \infty} a_{m,n} = 0$$

↓
 $\liminf_{n \rightarrow \infty} \left(\sum_m a_{m,n} \right) = 1$

~~1/2, 1/4, 1/8, 1/16, etc.~~