4.2 Some Simple Results

<u>Lemma</u>

Let R be any ring, and let $a, b \in R$. Then:

a. $a0 = 0a = 0$ ← Each of these zeros denote the zero element of the ring

b. $a(-b) = (-a)b = -(ab)$

c. $(-a)(-b) = ab$

If R has a multiplicative identity 1, then we also have

d. $(-1)a = -a$ ← Here -1 is the additive inverse of 1: $1 + (-1) = 0$

<u>Proof:</u>

a. We have $0 = 0 + 0$, so by the distributive laws

$$0a = (0+0)a = 0a + 0a.$$

Subtract $0a$ from both sides to get $0 = 0a$.

b, c, d. Exercises.

Exercise

If R is a ring and $a, b \in R$, then

$$(a+b)^2 = a^2 + ab + ba + b^2$$

In general, $ab \neq ba$