C. Heil

Below is a quick list of some of the highlights from the sections of the text that we have covered. You should be understand and be able to use or apply each item as appropriate. THIS LIST MAY NOT BE COMPLETE! Add additional items to it as you study for the final exam. If there's anything you are not sure of, ask! It's very important to be able to do calculations, but you also need to know WHAT things are and WHY they are what they are.

Special notes

Our final exam is Tuesday, December 11, 11:30 a.m.–2:20 p.m., in the lecture room (Weber SST Room 2). Arrive early so that we'll be able to start on time.

The final is worth 75 points. No books or calculators allowed on the final exam.

SHOW YOUR WORK on the exam! To get partial credit, we have to be able to figure out what you are doing! The answer at the end is the LEAST important part of the problem, it's the WORK that is important! WRITE CLEARLY!

The final is comprehensive, but at least 50% of the points will be drawn from the material covered since Exam 3. That is, the majority of points will be drawn from Sections 5.4 and later—but there will still be a substantial number of points drawn from earlier material!

I will be away the last week of class (December 3-7). During that week, the TA's will have their usual office hours. Additionally, the following TA's will have office hours open to students from all three of our sections, please feel free to go to any of these office hours:

Kevin Khan	Skiles 153	2-4 p.m., Wed, Dec 5
Meeti Shah	Skiles 230	2-3 p.m., Tues, Dec 4 and Thurs, Dec 6

I will be available most of Monday of finals week.

The homework assignments for the semester, solutions to the old quizzes and exams for the semester, and a practice final exam are available in the following directory:

`ftp://ftp.math.gatech.edu/pub/users/heil/1501`

Chapter 1

The material in this chapter is *background* that you should already be familiar with. The following things are especially important.

a. Know how to work with inequalities, especially ones involving absolute value.
b. What is a function? Know the meanings of domain and range and how to find them. Know how to work with compositions $f \circ g$, including finding the formula and the domain.
c. The six trig functions are extremely important. Know what all of them are. Review pages 42–45 in the text carefully.
d. What does the graph of each of $\sin x$, $\cos x$, $\tan x$, $\cot x$, $\sec x$, and $\csc x$ look like?
e. What are the values of each of those functions at $x = 0, \pi/6, \pi/4, \pi/3, \pi/2$? (If you know the values of $\sin x$ and $\cos x$ at those points, you can get all the others.)
f. Can you use basic facts about changing the graph of a function to plot the graphs of functions like $\sin 2x$, $3\cos x$, $\cos(x - 3\pi/2)$, $|\sin \pi x|$, etc.?
g. Know basic trig identities, especially

$$\sin^2 x + \cos^2 x = 1$$
$$\tan^2 x + 1 = \sec^2 x$$
$$\sin 2x = 2 \sin x \, \cos x$$

Chapter 1 Homework

1.2 # 3, 5, 9, 21, 23, 27, 31, 37, 45, 49, 57
1.3 # 5, 7, 9, 11, 15, 19, 23, 31, 35, 39, 41, 43, 47, 53, 55, 59
1.4 # 5, 9, 13, 21, 27, 29, 31, 35, 41, 45, 51, 59
1.5 # 3, 9, 11, 13, 19, 27, 37, 43, 45, 47, 59, 69
1.6 # 12, 16, 17, 26, 33, 53, 61, 69
1.7 # 11, 14, 27, 36, 45, 51, 54, Project 1.7

Chapter 2

2.1 The idea of limit. Main points:

a. What is the intuitive definition of a limit? (See p. 60.)
b. Notation for limits.
c. How to calculate limits.
d. One-sided limits.

2.2 The definition of limit. This is the section containing the ε-δ definition of limit. I will not test this material on the final exam.

2.3 Some limit theorems. Main points:

a. Limits are unique.
b. IF $\lim_{x\to a} f(x)$ and $\lim_{x\to a} g(x)$ exist then

$$\begin{aligned}
\lim_{x\to a}[cf(x)] &= c\lim_{x\to a} f(x) \\
\lim_{x\to a}[f(x)+g(x)] &= \lim_{x\to a} f(x) + \lim_{x\to a} g(x) \\
\lim_{x\to a}[f(x)g(x)] &= \left(\lim_{x\to a} f(x)\right)\left(\lim_{x\to a} g(x)\right) \\
\lim_{x\to a}\frac{f(x)}{g(x)} &= \frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)} \quad \text{if } \lim_{x\to a} g(x) \neq 0
\end{aligned}$$

But the limit of $f(x)+g(x)$ or $f(x)g(x)$ or $f(x)/g(x)$ CAN exist even though the limit of $f(x)$ or the limit of $g(x)$ doesn't exist!

2.4 Continuity. Main points:

a. What is the definition of continuity?
b. If f and g are both continuous at x, then cf, $f+g$, fg and f/g are all continuous at x (for f/g you also require that $g(x) \neq 0$).
c. Continuity from the right and from the left.
d. What is the difference between f being continuous at $x = c$ and f having a limit as $x \to c$?

2.5 Pinching Theorem and trigonometric limits. Main points:

a. What is the pinching theorem and how do you use it?
b. Trigonometric limits. Know the facts

$$\lim_{x\to 0}\frac{\sin x}{x} = 1 \qquad \text{and} \qquad \lim_{x\to 0}\frac{\cos x - 1}{x} = 0$$

and be able to use these to compute other limits.

2.6 Properties of continuous functions. Main points:

a. Intermediate Value Theorem: what it is and how to use it.
b. Boundedness Theorem: what it is and how to use it.

Chapter 2 Homework

2.1 # 1, 5, 9, 11, 13, 17, 21, 25, 27, 31, 35, 37, 41, 43, 47
2.2 # 1, 3, 5, 11, 13, 17, 19, 21, 23, 27, 29, 31, 33, 41-45, 51 (won't test on this section)
2.3 # 1, 3, 5-37 odd, 39, 41, 43, 45, 47, 49, 51, 55
2.4 # 1, 5, 7, 11, 15, 17, 19, 23, 27, 29, 33, 35, 41, 43, 51
2.5 # 1-15 odd, 19-31 odd, 39-43
2.6 # 1, 3, 5, 9, 15, 19, 21, 23, 25, 27, 29, 31, 33

Chapter 3

3.1 Derivatives. Main points:

a. Definition of the derivative. Meaning of secant lines and tangent lines.
b. How to use the definition of the derivative to compute the derivative of functions.
c. If f is differentiable at $x = c$, then f is continuous at $x = c$. But f could be continuous at $x = c$ WITHOUT being differentiable at $x = c$! Give an example!
d. How to tell from a sketch of the graph of a function where it is differentiable.
e. How to draw an approximate sketch of the derivative of a function. Important!

3.2 Differentiation formulas. Main points:

a. Formulas for the sum, product, and quotient of a derivative. KNOW THE PRODUCT RULE AND THE QUOTIENT RULE!!

3.3 $\dfrac{d}{dx}$ notation, higher derivatives. Main points:

a. How to use $\frac{d}{dx}$ notation, especially with the chain rule.
b. Notations for higher derivatives.
c. Derivative of x^n for integer n.

3.4 Derivative as a rate of change. Main points:

a. Why is rate of change a derivative?
b. Motion in a straight line, velocity, acceleration. When is it moving right or left, speeding up or slowing down?
c. Motion under gravity.

3.5 Chain Rule. Main points:

a. What is the chain rule in both Newton and Leibniz notation.
b. How to use the chain rule. EXTREMELY IMPORTANT!!

3.6 Trig functions. Main points:

a. What are the derivatives of ALL SIX trig functions?
b. Combine product rule, quotient rule, and chain rule with trig functions.

3.7 Implicit differentiation. Main points:

a. What does it mean for a function to be defined implicitly?
b. How do you find the derivative of a function if it is only given implicitly? Answer: differentiate both sides of the equation that gives the function implicitly, then solve for dy/dx.

3.8 Rates of change per unit time. Word problems. Main points:

a. If two or more quantities are related, then their derivatives will be related too.
b. To solve a related rates problem: draw a picture, find out what what are the functions and what are the variables, find out what quantities you KNOW (especially relationships between the functions), differentiate both sides of the relationships, solve for the derivative you want, and finally figure out what that derivative is AT the time or place that you want it.

Chapter 3 Homework

3.1 # 1, 5, 7, 9, 11, 13, 17, 25, 26, 27, 31, 33, 37, 39, 41, 43, 47, 49, 51, 55, 65
3.2 # 3, 5, 9, 13, 17, 19, 23, 25, 29, 37, 43, 47, 55, 61, 67, 73
3.3 # 3, 5, 7, 11, 15, 17, 19, 23, 27, 29, 33, 35, 39, 43, 51
3.4 # 1, 5, 9, 13, 15, 17, 21, 23, 25, 27-36, 37, 41, 45, 47, 51, 55, 57
3.5 # 1, 5, 7, 9, 11, 17, 19, 23, 25, 27, 29, 35, 37, 41, 45, 47, 49, 55, 61, 65, 67, 73
3.6 # 1, 3, 5, 7, 9, 11, 15, 17, 19, 25, 27, 29, 35, 39, 41, 45, 49, 51, 53, 61, 63, 65, 69, 71
3.7 # 3, 5, 7, 9, 11, 15, 17, 19, 25, 27, 31, 39, 41, 51, 59
3.8 # 1-39 odd

Chapter 4

4.1 The Mean-Value Theorem. Main points:

a. What does the Mean-Value Theorem say?
b. How do you use it in problems?

4.2 Increasing and decreasing functions. Main points:

a. Definition of increasing and decreasing.
b. IF f is differentiable AND $f'(x) > 0$ for $x \in (a, b)$, then f is increasing on (a, b).
c. IF f is differentiable AND $f'(x) < 0$ for $x \in (a, b)$, then f is decreasing on (a, b).
d. IF f is differentiable AND $f'(x) = 0$ for $x \in (a, b)$, then f is constant on (a, b).
e. Be able to determine where a function is increasing, decreasing, or constant.
f. Given f, how to draw an approximate sketch of f'.
g. Given a sketch of f', how to get information about f.

4.3 Local Extreme Values. Main points:

a. Definitions of critical points, extreme points, local max, local min.
b. The First Derivative test and how to use it. A function must CHANGE from increasing to decreasing to have a local max, and must CHANGE from decreasing to increasing to have a local min.
c. The Second Derivative test: what is it and how do you use it?
d. Be able to determine critical points and tell which of those are extreme points and what kind of extreme points they are (max or min).

4.4 Endpoint and absolute extreme values. Main points:

a. Definition of endpoint extreme values.
b. Definition of absolute extreme values.
c. Be able to find ALL the extreme values of f on a closed interval $[a, b]$—look for critical points within (a, b) and test them, AND test the endpoints. YOU MUST CHECK THE ENDPOINTS!

4.5 Max/min word problems. Main points:

a. Determine the function that you want to find the max or min of.
b. Eliminate variables until you have a function of one variable alone.

c. Find ALL the extreme values of that function—you MUST test the endpoints as well (unless the domain of the function in question has no endpoints). CHECK THE ENDPOINTS!
d. Answer the question asked—give the quantity asked for, whatever it is.

4.6 Concavity and inflection points. Main points:

a. Definitions of concave up, concave down, inflection points.
b. The concavity MUST CHANGE in order for a point to be an inflection point. An inflection point is NOT just where $f''(x) = 0$!! The concavity MUST CHANGE!!
c. f is concave up where f' IS INCREASING, and this happens when $f''(x) > 0$.
d. Given f, how to draw an approximate sketch of f' and f''.
e. Given a sketch of f', how to get information about f and f''.
f. Given a sketch of f'', how to get information about f and f'.

4.7 Vertical and horizontal asymptotes. Main points:

a. Be able to find the vertical and horizontal asymptotes.
b. For horizontal asymptotes, also be able to determine whether the function is ABOVE or BELOW the asymptote.
c. For vertical asymptotes, also be able to determine whether the function is heading to $+\infty$ or to $-\infty$ as x approaches the asymptote line.
d. Use this information to sketch the function.

4.8 Curve sketching. Main points:

a. Be able to find where f is increasing, decreasing, find the critical points and extreme points, determine the concavity and inflection points, and find the asymptotes.
b. Use this information to sketch the graph of f.

Chapter 4 Homework

4.1 # 3, 7, 9, 11, 13, 15, 19, 21, 23, 29, 33, 37, 43, 44, 45, 46
4.2 # 1-13 odd, 17, 19, 23, 25, 27, 29, 31, 37, 39, 41-46, 53
4.3 # 3, 5, 9, 13, 15, 17, 19, 21, 25, 27, 29, 33, 39, 41
4.4 # 1, 5, 9, 11, 17, 19, 21, 25, 29, 31, 33, 39, 43
4.5 # 3, 5, 7, 9, 13, 15, 19, 21, 23, 27, 29, 33, 35, 39, 43, 49, 53
4.6 # 1, 2, 5-27 odd, 31, 37
4.7 # 1, 2, 3, 5, 9, 13, 17, 19, 21, 23, 31, 35, 37, 45
4.8 # 5, 9, 11, 13, 23, 25, 29, 33, 41, 49, 55

Chapter 5

5.1 Definite integrals. Main points:

a. Know what each of the following terms means:
   partition P,
   subintervals $[x_{i-1}, x_i]$,
   length of the subinterval Δ_i,
   size (norm) $\|P\|$ of P,
   min m_i,
   max M_i,
   lower sum $L_f(P)$,
   upper sum $U_f(P)$,
   points x_i^*,
   Riemann sum $S^*(P)$.

b. Definition of the definite integral $\int_a^b f(x)\,dx$: it is the unique number (if one exists) such that $L_f(P) \le \int_a^b f(x)\,dx \le U_f(P)$ for EVERY partition P. Each lower and upper sum therefore gives an estimate of the value of the integral.
c. How do the Riemann sums $S^*(P)$ compare to the lower and upper sums and to the integral $\int_a^b f(x)\,dx$? What happens to the lower, upper, and Riemann sums as the size $\|P\|$ of the partition decreases to zero?
d. Given a function f and specified partition P, be able to compute $L_f(P)$, $U_f(P)$, $S^*(P)$ and to sketch the boxes involved in computing those sums.

5.2 $F(x) = \int_a^x f(t)\,dt$. Main points:

a. If f is continuous on $[a,b]$, then F is one antiderivative of f. This means that $F'(x) = f(x)$, or:
$$\frac{d}{dx}\int_a^x f(t)\,dt \;=\; f(x).$$
b. Any other antiderivative of f has the form $G(x) = F(x) + C$ where C is a constant.
c. Know basic properties of F: it is differentiable on (a,b) (even if f isn't differentiable!), $F(a) = 0$, if $f(x) \ge 0$ everywhere then $F(x)$ is increasing, etc.
d. Know integration formulas such as
$$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx, \qquad \int_a^b f(x)\,dx = -\int_b^a f(x)\,dx.$$
e. Do computations with variations on $F(x)$ such as $\int_a^{x^3} f(t)\,dt$. Need to apply the CHAIN RULE here!

5.3 Fundamental Theorem of Calculus (FTC). Main points:

a. What does the FTC say?
b. Use the FTC to compute definite integrals involving simple functions such as x^r, $\sin x$, $\cos x$, $\sec^2 x$, $\sec x\,\tan x$, and more complicated ones such as $\int_0^1 (x^3+1)\,3x^2\,dx$. These are easy cases of the more complicated u-substitutions that we do later.

5.4 Area problems. Main points:

a. Given some curves, be able to sketch the region enclosed by them.
b. Then be able to set up and work out the integral the gives the area enclosed by the curves.
c. Be especially careful if the curves cross: then you have to break up into subregions, and compute a separate integral for each subregion.

5.5 Indefinite integrals. Main points:

a. $\int f(x)\,dx$ denotes a generic antiderivative of $f(x)$.
b. Therefore, if $F(x)$ is any ONE antiderivative of $f(x)$, then $\int f(x)\,dx = F(x) + C$ gives the generic form of all the possible antiderivatives of $f(x)$.

5.6 u-substitutions. Main points:

a. You want to find a u so that your integral can be written as $\int f(u)\,du$. Then if f is nice enough, you can work out the integral in terms of u and substitute back for u at the end.
b. PRACTICE, PRACTICE, PRACTICE until you can see how to choose u!
c. The table on p. 302 is a good summary of some basic formulas.

5.7 Additional properties. Main points:

a. If $f(x) \ge 0$ everywhere then $\int_a^b f(x)\,dx \ge 0$.

b. $\left|\int_a^b f(x)\,dx\right| \le \int_a^b |f(x)|\,dx$.

Chapter 5 Homework

5.1 # 1-9 odd, 11-17, 21, 23, 25, 35

Typos: #13, 14 should say "Repeat exercise 12", #35c should say "Use exercise 33"

5.2 # 1-29 odd, 33

5.3 # 1-35 odd, 37, 41, 45, 49, 51, 53, 55, 57

5.4 # 3, 5, 9, 11, 15, 19, 23, 25, 27, 28, 29, 30, 33, 35

5.5 # 1, 5, 9, 13, 15, 17, 23, 27, 31, 37, 49

5.6 # 1, 7, 9, 11, 17, 19, 23, 25, 29, 33, 37, 39, 41, 45, 47, 49, 53, 61, 65, 69, 71, 73, 75

5.7 # 1, 3, 5, 7, 9, 11, 13, 15, 17, 23

Chapter 6

6.1 More on area. Main points:

a. Really understand the connection between an integral and a Riemann sum.
b. Given a region bounded by some curves, be able to partition either the x-axis or the y-axis and find the corresponding Riemann sum that approximates that area. Then by looking at the formula for the Riemann sum, be able to determine the integral that represents the true area.

6.2 Volumes by discs and washers. Main points:

a. Given a solid, be able to partition an axis and then find the volume of one "slice" of the solid.
b. For solids of revolution, such a slice will be a disc or a washer (this section) or a shell (Section 6.3).
c. For something that isn't a solid of revolution, you need to think carefully about what the volume of the slice will be. See especially problems 27–33 in this section. Important!
d. Get the Riemann sum approximation to the volume of the solid by adding up the volumes of all the slices.
e. From the Riemann sum approximation, be able to determine the integral that represents the true volume of the solid.
f. The MAIN POINT is to understand how to do these problems by Riemann sums. Formulas such as equation (6.2.3) on page 331 follow from this—you need to understand Riemann sums, not memorize those formulas. Besides, those formulas ONLY apply to solids of revolution.

6.3 Volumes by shells. Main points:

a. Given a solid of revolution, be able to partition the axis and find the volume of one "slice" of the solid.
b. If the slice is a shell, be able to find the volume of that shell, and then by choosing x_i^* (or y_i^* if partitioning the y-axis) to be the midpoint of the ith subinterval, turn that volume into something that involves only x_i^* and Δx_i (or y_i^* and Δy_i if partitioning the y-axis).
c. Then find the Riemann sum and then find the integral that gives the true volume of the solid.

6.5 Work. Main points:

a. Work is force times distance. If the force is from gravity (as in all the problems that we will do), then force = weight.
b. In English units, weight = pounds (lbs). In Metric units, weight = mass times acceleration due to gravity, with mass given in kilograms (kg) and the acceleration due to gravity being 9.8 m/sec^2.
c. Weight = volume times weight per unit volume. For example, water weighs 62.5 lbs per cubic foot, so x ft^3 of water weighs $62.5x$ lbs.

d. Much as in the volume problems, partition your object vertically, then find the work to move each slice of the object. That work is the weight of that slice times the distance that slice moves. Add up the work for each slice to get the Riemann sum approximation to the total work. That leads you to the integral that represents the true work performed on the entire object. The big difference between these problems and volume problems is that you also have to take into account the distance that the slice moves.

Chapter 6 Homework
6.1 # 3, 7, 11, 17, 21, 23, 25, 29, 35, 37, 44, 45
6.2 # 5, 11, 13, 19, 25, 27, 29, 31, 33, 39, 43, 47
6.3 # 3, 5, 9, 11, 15, 17, 25-30, 39
6.5 # 1, 3, 13, 15, 17, 19, 21, 25, 27

Chapter 7

7.1 One-to-one functions and inverse functions. Main points:

a. A function f is 1-1 if $f(x_1) \neq f(x_2)$ whenever $x_1 \neq x_2$.
b. To test for 1-1, use the horizontal lines test.
c. A function that is also increasing or always decreasing on a connected domain is sure to be 1-1.
d. A 1-1 function f has an inverse function f^{-1}. The main property of the inverse function is that

$$f(f^{-1}(x)) = x \qquad \text{and} \qquad f^{-1}(f(x)) = x.$$

e. Given $f(x)$, be able to find a formula for the inverse function. Be able to find the domain of the inverse function.
f. The graph of $y = f^{-1}(x)$ is found by taking the mirror image of the graph of $y = f(x)$ about the line $y = x$.
g. The derivatives of f and f^{-1} are related:

$$(f^{-1})'(b) = \frac{1}{f'(a)} \qquad \text{if } b = f(a).$$

7.2 Logarithms. Main points:

a. $\log_a x$ is the exponent that a must be raised to in order to get x. That is,

$$y = \log_a x \iff a^y = x.$$

In other words: a^x and $\log_a x$ are INVERSE FUNCTIONS. If $f(x) = a^x$ then $f^{-1}(x) = \log_a x$!

b. Some important properties:

$$\begin{aligned}
\log_a a^x &= x \\
a^{\log_a x} &= x \\
\log_a a &= 1 \\
\log_a 1 &= 0 \\
\log_a xy &= \log_a x + \log_a y \\
\log_a x/y &= \log_a x - \log_a y \\
\log_a 1/x &= -\log_a x \\
\log_a x^y &= y \log_a x \\
\text{domain}(\log_a x) &= (0, \infty)
\end{aligned}$$

c. Natural logs are logs with base $a = e \approx 2.818\ldots$. Notation: $\ln x = \log_e x$. Write down all the properties given in part b above for the function $y = \ln x$!

d. Additionally, $\ln x$ is an antiderivative of $1/x$:

$$\ln x \;=\; \int_1^x \frac{1}{t}\,dt,$$

i.e., $\ln x$ is the area under the curve $y = 1/t$ between 1 and x. Consequently,

$$\frac{d}{dx}\ln x \;=\; \frac{1}{x}, \qquad x > 0.$$

7.3 More on natural logs. Main points:

a. While $\ln x$ is only defined when $x > 0$, the function $\ln|x|$ is defined for all $x \neq 0$ and is differentiable for all $x \neq 0$.

b. $\dfrac{d}{dx}\ln|x| = \dfrac{1}{x}$. Applying the chain rule, we therefore get:

$$\frac{d}{dx}\ln|u| \;=\; \frac{1}{u}\frac{du}{dx} \qquad \text{and} \qquad \int \frac{1}{u}\,du \;=\; \ln|u| + C$$

Be able to use these!

c. The formulas in the boxed equation (7.3.6) on p. 387 will be supplied to you on the final exam, but you need to be familiar with them and know how to use them.

d. There will be no problems on logarithmic differentiation (see pages 388-390 of the text).

7.4 e^x. Main points:

a. e^x and $\ln x$ are inverse functions.

b. $\dfrac{d}{dx}e^x = e^x$. By the chain rule, we then get:

$$\frac{d}{dx}e^u \;=\; e^u\frac{du}{dx} \qquad \text{and} \qquad \int e^u\,du \;=\; e^u + C$$

Be able to use these!

7.6 Exponential growth and decay. Main points:

a. The solution to

$$f'(x) = kf(x) \qquad \text{is} \qquad f(x) = Ce^{kx}$$

where C is a constant. In fact, $C = f(0)$.

b. If $k > 0$ then this is *exponential growth*, if $k < 0$ then it is *exponential decay*.

c. Be able to do word problems about population growth, radioactive decay, etc.

Chapter 7 Homework

7.1 # 5, 9, 13, 17, 21, 28-31, 37, 39, 51
7.2 # 1, 11, 13, 15, 17, 19, 23, 25
7.3 # 3, 7, 11, 13, 17, 21, 25, 31, 35, 41, 45, 57, 65
7.4 # 3, 5, 7, 11, 13, 17, 19, 21, 25, 29, 31, 33, 35, 37, 39, 41, 43, 51, 61, 65, 67
7.6 # 5, 7, 9, 11, 15, 17, 21, 25

Chapter 8

8.1 Review of integration formulas. Main points:

a. You should memorize equations 1–4, 6–7, and 12–15 on page 441 of the text.

b. Formulas 8–11 on page 441 of the text will be supplied to you, but you need to be familiar with them.

8.2 Integration by parts. Main points:

a. Be familiar with the integration by parts formulas in both the indefinite and definite forms:

$$\int u\,dv \;=\; uv - \int v\,du$$

and

$$\int_a^b u\,dv \;=\; uv\Big|_a^b - \int_a^b v\,du \;=\; u(b)v(b) - u(a)v(a) - \int_a^b v\,du$$

b. PRACTICE, PRACTICE, PRACTICE until you're able to see how to choose u and dv.
c. Once u and dv are chosen, you find du and v and use the integration by parts formulas above.
d. Sometimes you have to do integration by parts twice, then solve for the integral you want.
e. Be able to tell whether an integral needs to be done by u-substitution or by integration by parts.

Chapter 8 Homework

8.1 # 1, 3, 5, 7, 9, 11, 13, 17, 19, 21, 27, 29, 31, 35, 37

Hints: $\ln x^3 = 3\ln x$, $\tan^2 x + 1 = \sec^2 x$, $e^{1/x} = e^{(x^{-1})}$

8.2 # 3, 5, 7, 9, 13, 17, 19, 21, 23, 25, 29, 39, 41

Hints: $\ln x^{1/2} = (1/2)\ln x$, #7 do integration by parts twice, #13 take $dv = dx$

Complex Numbers

Complex number handout. Main points:

a. The imaginary unit is $i = \sqrt{-1}$. So $i^2 = -1$.
b. A complex number has the form $z = a + bi$ where a, b are real numbers. Know what the complex plane is.
c. Know how to find the real and imaginary part of a complex number: if $z = a+bi$ then $\mathrm{Re}(z) = a$ and $\mathrm{Im}(z) = b$.
d. The modulus or absolute value of $z = a + bi$ is $|z| = \sqrt{a^2 + b^2}$.
e. The argument of z is the angle from the real axis to z. Write $\arg(z)$ for the argument of z. It must be in the range $0 \le \arg(z) < 2\pi$.
f. Given z, be able to find $|z|$ and $\arg(z)$. Given $|z|$ and $\arg(z)$, be able to find z.
g. The polar representation of z is $z = (r\cos\theta) + (r\sin\theta)i$ where $r = |z|$ and $\theta = \arg(z)$.
h. The complex conjugate of $z = a + bi$ is $\bar{z} = a - bi$.
i. Be able to add and multiply complex numbers.
j. Be able to multiply complex numbers in the polar representation form. Remember that $|zw| = |z|\,|w|$ and $\arg(zw) = \arg(z) + \arg(w)$.

Complex number handout Homework

1, 3, 5, 7, 9, 13, 15, 16, 21, 23, 25, 27, 29, 33, 35, 37, 39, 47